

## **MODELING OF WASTEWATER TREATMENT PLANT DESIGN FOR PULP AND PAPER INDUSTRY: A REVIEW**

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### **ABSTRACT**

Pulp and paper industry is responsible for large discharge of highly polluted effluents. The modeling environment of wastewater treatment plant includes several mathematical techniques, interactive graphic displays, and user-friendly interfaces. In view of the multiple factors and parameters affecting the wastewater treatment plant design, a trial-and-error design procedure is commonly used until each treatment unit's design has been adjusted to the targeted effluent and sludge characteristics which are time consuming and complex. However, in order to take advantage of the possibilities offered by the software's, it is necessary to create the suitable software adjusted to the specific requirements of their application area. Considering these factors and necessity this paper took the review of the existing literature of the modeling processes and various software's available for the design of wastewater treatment plants for pulp and paper industry.

**KEYWORDS:** Pulp and Paper, Modeling, White Box, Mathematical Software, Wastewater Treatment

### **INTRODUCTION**

Wastewater treatment is a widely used process for both residential and industrial purposes. In analyzing wastewater treatment plant design (WTPD) problems, physical, chemical and biological parameters are to be considered. In view of the multiple factors and parameters affecting complete wastewater treatment plant design, a trial-and-error design procedure is commonly used until each treatment unit's design has been adjusted to the targeted effluent and sludge characteristics which are time consuming and complex. Several wastewater treatment software tools are available in the market such as Mathematica, Maple, MATLAB, Mathcad etc. Such software tools provide vast capabilities for the modelling and optimization tools for engineers and researchers.

Amongst Indian industries, pulp and paper industry is the greatest industrial polluter in terms of wastewater volumes and organic discharges. With the ever-increasing social awareness of environmental protection issues, the proper operation and control of wastewater treatment plants (WWTPs) have come under scrutiny. Optimization at every stage of design a WWTP demands systematic investigations and the determination of control strategies to minimize multiple objective criteria. Although this industry discharges gaseous, liquid and solid wastes, pollution of the aquatic ecosystem is the major problem as large volumes of wastewater are generated while producing paper.

An overview of the most frequently applied models for the design of wastewater treatment plants especially for paper and pulp industries is provided.

## LITERATURE REVIEW

Literature reviews of the wastewater treatment plant design especially for pulp and paper mill is mainly divided in two parts are as follows

- Computational tools
- Mathematical computing tools

## COMPUTATIONAL TOOLS

**Gass S. and Baldano J. M. (1992)** used **SIMTAR** application which was based on CAD/CAE system for design and economic evaluation of wastewater treatment plant. It allows the study of numerous design alternatives, including both technical and economic issues. In this system all the stages involved in the design of WTP's - calculation of equipment dimensions, cost estimation, and planning drawing up can be accomplished. The data input is done interactively by means of a graphical interface, which permits generation of the simulation diagram and the treatment process flow-sheet. To test the performance of this CAD/CAE system, design of some WTP's performed with it has been compared with those proposed by several engineering companies. It can be stated that the equipment sizes and the costs computed by "SIMTAR" fit quite well with those calculated by the engineering firms.

**Sundara Kumar K (2011)** states that one of the way for easy design with C++ language for repeat the design calculations with different sets of input data and optimal size of the system very easily. A computer program in C++ has been written for interactive computer aided design of wastewater treatment plant. A WTP located in Nesapakkam was considered for case study. Author has used the existing plant data for verifying the software's authenticity and concluded that the program works well and it can be used for the design of any WTP which is having activated sludge process as biological process with relevant input data. The program can also be used to check the design details of an existing plant to know the expansion works needed for increased hydraulic and organic loadings occurring in future.

**Khitoliya R K et al (2006)** studied **ETPSOFT** software which is based on C++ language and basically innovate for design of effluent treatment plants. Author states some advantages of ETPSOFT like Menu and submenu to select a particular unit for design, Hydraulic design procedure according to standard references, an option to input desired design data, an alarm warning if value of any particular parameter entered exceeds or falls short of range usually practiced, a calculator available throughout the program run primarily to aid the user in converting any data to the units of measurement needed in the program. As environmental point of view ETPSOFT does include various advanced treatment units like ion exchange, reverse osmosis, sludge thickeners, sludge centrifugal units etc. could be appended to this software making it a complete design.

**Carmen Gabaldo'n et. al. (1998)** performed detailed investigation about **DATAR** software which is based on Microsoft Visual Basics 3.0 programming language. This software package has been developed for automated design of WTP. A user-friendly environment has been implemented to facilitate design tasks, allowing rapid evaluation of different alternatives as well as performing sensitivity analyses. The paper focuses on illustrating the potential uses of the software rather than describing the implemented mathematical models. The software incorporates the main wastewater and sludge processes including different types of screening, grit removal, chemical coagulation, sedimentation, biological waste water treatment, effluent disinfection, sludge concentration and stabilization, also include nitrification–denitrification processes.

Mathematical models describing treatment processes have been formulated taking into account the variation in waste quality parameters for two different climatic conditions (winter and summer seasons) at each treatment element. The developed software has been successfully applied to the simulation of two full-scale wastewater treatment plants. Authors concluded that the software developed is an effective tool both in analyzing different design and upgrading alternatives and predicting process performance.

**Sriwiriya T (1999)**, develops Integrated Film Activated Sludge (**IFAS**) process computer based mechanistic model which can be used as a tool by scientist and an engineer to optimized their designs and to troubleshoot a full scale treatment plant. It was also developed to reduce the cost of additional facilities required to complete year round nitrification in the design of new or retrofit wastewater treatment plants. IFAS enables the steady-state simulation of nitrification-denitrification processes as well as carbonaceous removal in systems utilizing integrated media. IFAS was written for IBM-PC compatible micro computers in Borland C++ Builder suite. IFAS program was written and compiled on Windows 98 and incompatible with Windows 95/98 or Windows NT. Many functions were added to the program to make it user-friendly and interactive to users. IFAS allows the user to enter the specific operating and design data, in addition to the wastewater characteristics.

**Zeidana A. et. al. (2003)**, simulated a wastewater treatment process by using **BioSys** a compuer based software which is a powerful technique for optimization and improvement of current and under-construction wastewater facilities. The software is useful for optimizing and improving new facilities and existing plants using simulation. A special capability is provided by BioSys for the simulation of UV disinfection unit. It is a significant tool for the design and optimization of the UV disinfection units. BioSys is also useful for selecting the best plant layouts and design alternatives, evaluating several feed scenarios, predicting the effect of adding or removing a unit for maintenance, predicting the effect of rainfall conditions, controlling mass balances and evaluating new designs.

## MATHEMATICAL TOOLS

**GPS-X** is the most advanced tool available for the mathematical modeling, simulation, optimization and management of wastewater treatment plants. The user-friendly drag and drop interface and comprehensive database of unit processes allows users to quickly and easily assemble a treatment plant model, enter characterization data, and run simulations. It can develop and optimize advanced control schemes, predict effluent quality under varying conditions, conduct planning and capacity analysis. Summarized output data is available to be viewed, copied and / or printed with a single mouse click. Custom report templates can be designed for specific jobs, it having open modelling platform, with open-source models. The GPS-X simulation environment allows users to write / edit / change models, via the advanced continuous simulation programming language (ACSL). It gives feature like addition of any new calculations / algorithms / variables to model layout. (**GPS-X User Guide**).

The **GPS-X Hybrid Model**, modelling software was employed in the study by **Carol L A Maas et al (2005)**. The hybrid layout employs the “Mantis” model, a modified version of ASM1 which incorporates temperature dependent kinetic coefficients, aerobic denitrification and additional growth processes that allow nitrate to be taken up as a nutrient under conditions of low ammonia. Mantis is used for calculation of all biochemical conversions in the suspended growth and biofilm. The primary objective of the study was to determine the model’s performance in the absence of calibration information. Average influent wastewater quality data and operating conditions extracted from the literature references were entered into the model. The model was run at steady state and output data recorded. The impact of process variables

on performance was evaluated by modifying operational and influent parameters in the model. The model accurately predicted the average improvement in nitrification efficiency observed following an upgrade to integrated fixed film/activated sludge (IFAS) in the Waterdown STP at default parameters.

**Sofia F P (2014)** used **GPS-X** for modelling of wastewater treatment plant for Portucel Soporcel Mill, a pulp and paper industry. This program has a clear-cut graphical interface and uses a specialized translator that converts the graphical process into material balance equations, based on dynamic models. These models allow, besides the kinetic description of the treatment process carried out at the WWTP, to simulate new scenarios towards the study of critical parameters for the process as well as optimization and control of the WWTP. The design of the WWTP in GPS-X involved not only the collection of physical data concerning the unit processes but also the collection of historical data about the WWTP performance over the last three years.

**Hu et al (2008)** concluded that current density, initial pH, electrolyte species, initial concentration of dye produce the effects upon the removal of C. I. Reactive Red 241 by electrocoagulation in regards to decolorization. From the results, it was determined that dye removal was a first order reaction, where the COD removal could be determined using the artificial neural network (ANN) and response surface method (RSM) models.

An artificial neural network (ANN)-based predictive model was developed by **Saraswathi R. and Raggul N. (2014)**, to explore the feasibility of using the predominant microorganism present in the paper and pulp mill wastewater and for the assessment of the performance of paper and pulp ETPs and to determine the thickener area of the clarifier by correlating process control parameters. A feed-forward algorithm consisting of one input layer, one hidden layer and one output layer, all connected with no feedback connections, was applied. The feed-forward NN was trained for a given value of  $\eta$  and different values of  $R = 0.25-1.0$  and the trained network was tested with the test data. The quality of the match between the ANN model values and experimentally measured values was verified with the mathematical model by using SPSS software.

**Krist V. Gernaey (2004)**, focused on the developments of activated sludge models, mainly the family of activated sludge models (ASM) developed by the International Water Association (IWA) and the metabolic model developed at the Delft University of Technology (TUDP model). Activated sludge modeling and simulation are widely applied. Learning, design and process optimization are the main application areas of white-box WWTP models. The introduction of the ASM model was of great importance in this field, providing researchers and practitioners with a standardised set of basis models. Artificial Intelligence (AI) methodologies and white-box models can interact in many ways. A white-box model calibration tool, an AI based WWTP design tool and a knowledge representation tool in the WWTP domain are potential applications where a fruitful interaction between AI methods and white-box models could be developed.

**OTTER**, typical software include operational decision support, works optimization, plant design and operator training. It models the occurrence, formation and removal of a wide range of water quality parameters, from general parameters such as turbidity and colour, organic parameters such as DOC, inorganic such as bromate through to pesticides and microbiological indices.

**Dudley J., et al (2008)** has selected OTTER as the basis of the framework for the new water treatment simulator to be developed as part of TECHNEAU. (Commission of the European Communities, Research Directorate-General:

Technology Enabled Universal Access to Safe Water). The developed simulator had incorporated aspects of existing OTTER software to simulate individual treatment processes or a complete treatment plant. Authors found that using OTTER at waterworks have been successful, but have highlighted the relatively large data requirements for successful calibration and use. The empirical nature of the coagulation and flocculation models has meant that the calibrated models could not be applied much outside the calibration region, restricting the degree of optimisation that could be studied.

General structure of the **EnviroPro** software consists of the graphical user interface, the process simulation module and the economic evaluation module. It uses a graphical interface to enhance the human /computer communication and reduce the learning period, resulting in a tool that is simple to use and easy to learn, even for occasional users with limited process design and environmental background. The process simulation module of EnviroPro assist the engineer to interactively develop and analyse integrated flowsheets for waste recycling, treatment and disposal processes. **Petiides D., et al (1998)** had carried out a simulation study by using EnviroPro model to track a fate of Volatile Organic Compound (VOC) and other chemicals in integrated wastewater treatment facilities. They had also evaluated the process modification and extensions necessitated by new regulations. The results of their analysis can act as incentives for pollution prevention strategies that reduce waste generation at the source and minimizes the need for investment in pollution control. They had concluded that the EnviroPro can play a role in educating students and engineers on how to design and operate processes within environmental constraints.

**WatPro** is a steady-state water treatment modelling program, with a focus on disinfection and disinfection by-products. WatPro is the premier water treatment simulator for predicting water quality based on specific treatment processes and chemical addition. WatPro uses raw water quality parameters such as pH, and design and operating characteristics of process tanks, to simulate plant operation. WatPro having features like modeling disinfection with chlorine dioxide, calculated turbidity reductions by process and whole facility, calculated specific ultra violet absorbance (SUVA) in raw water, new utilities (calculator, file recall, notepad), improved reporting and calibration options models ClO<sub>2</sub> disinfection, includes use of ferrous salts for removing residuals. WatPro has a user-friendly interface that allows a schematic of the water treatment plant to be easily configured within minutes. WatPro User Guide (**Hydromantis, 2004**).

MATrix LABoratory (**MATLAB**) was invented in late 1970s, since then it is well adapted to numerical experiments owing to the constantly evolving algorithms, built-in functions and m-files those are based on the standard libraries such as LINPACK and EISPACK. (**MATLAB Tutorial**)

**Jiayu KANG et al (2009)**, established a mathematical model for pH control in the biological wastewater treatment system and described as Modeling of pH Control System. On the basis of traditional proportional integral derivative (PID) control and cascade control are adopted to carry out simulation and comparison with **MATLAB**. Author mentioned about building the simulation model for pH cascade control system as model building of a main controlled object and an auxiliary controlled object in the cascade loops and selection of a main adjuster and an auxiliary adjuster in the cascade loops.

**Krist V. Gernaey (2004)**, made review on modeling of wastewater treatment plants (WWTP). White-box modeling is widely applied in this field, with learning, design and process optimization as the main applications. Researcher gives information about black-box; stochastic grey-box and hybrid models are useful in WWTP applications for prediction of the influent load, for estimation of biomass activities and effluent quality parameters. In this paper researcher concentrated on activated sludge model for simulation of waste water treatment plant. Learning, design and

process optimization are the main application areas of white-box WWTP models.

**Kao J. J. et. al. (1993)** develops a technique to create a prototype design environment to facilitate wastewater treatment plant design. This design environment includes several mathematical techniques, interactive graphic displays, and user-friendly interfaces. The interactive graphic displays and the user interfaces are intended to provide a user friendly environment for manipulating data, models, and solutions. Sensitivity analyses can be performed for models as well as parameters. The prototype was initially developed on an IBM PC AT and then converted to an Apollo workstation environment. The user interface of the prototype on the Apollo workstation is much simpler and easier to use. The program for the interface was written with **DIALOG** and **GMR2D**.

**Ellis K. V. and Tang S. L. (2008)** proposed model based on dynamic programming optimization models by introducing the quantity and quality of the influent, the effluent standards to be met, and the size and location of the site on which the plant is to be built. In this model, 20 parameters are identified and on them the selection of the optimal treatment alternative is based. The mathematical technique used in the model is called the analytic hierarchy process. The objective of this model is to prioritize a set of decision variables (i.e., treatment alternatives) so that the optimal one can be selected from the ranking of the resultant priority list. In assessing the relative weighting (or importance) of  $n$  alternatives, pairwise comparisons are used, which means that any particular alternative is not compared to all others simultaneously, but rather one at a time. If the pairwise comparisons of the relative weightings of the alternatives are known, the priority of the alternatives can be found by evaluating the priority vector of a matrix whose elements are the pairwise comparisons of the weightings. Later the decision variable matrices were constructed according to the same principle used in constructing the parameter matrix.

Whenever working with symbolic complex formulae is concerned, there is nothing like **Mathematica**, high-level programming tool with graphics support. Mathematica is split into two parts, the kernel and the front end. The kernel interprets expressions (Mathematica code) and returns result expressions. The front end provides a Graphical User Interface, which allows the creation and editing of notebook documents containing program code with pretty printing, formatted text together with results including typeset mathematics, graphics, Graphical User Interface components, tables, and sounds. All contents and formatting can be generated algorithmically or interactively edited (**Mathematica Journal**).

**M. Levstek et al (2005)**, carried out experimental analysis of steady and non-steady-state operational conditions in order to assess the reliability of mathematical simulations based on a modified Activated Sludge Model (ASM) that was successfully calibrated at the starting steady-state conditions. A mathematical model was built, according to ASM for activated sludge, including nitrogen removal. Calibration can be done with different calibration procedures. Calibration calculations, employing appropriate numerical methods, were performed using **Mathematica 5.0** at steady state according to a logical stepwise procedure. Researcher concluded that both, the dynamic model predictions, and the measured responses of the real system to substantial changes in the operational conditions, showed a return to the stationary state when the initial steady-state operational conditions were restored.

**Maple** is a commercial computer algebra system developed and sold commercially by Maple soft. Users can enter mathematics in traditional mathematical notation and can create custom interfaces. There is support for numeric computations, to arbitrary precision, as well as symbolic computation and visualization. Maple incorporates a dynamically typed imperative style programming language which resembles Pascal. The language permits variables of lexical scope (**Maple Solutions**). Maple that offers,

- intuitive smart and self document environment with math equation editor,
- task templates and interactive task assistants,
- 2-D and 3-D plotting and animations.
- code generation and
- compatibility to Excel, MATLAB, C, Java, and FORTRAN.

**Svetoslav Markov (2005)**, discussed the use of Computer Algebra System in the modelling, optimization and control of biotechnological processes because a main characteristic of biological systems is uncertainty, due to the use of simplified models, variation with time of parameters, unpredictable phenomena, inexact measurements, intrinsic sensitivity of the system, etc. Author explained that computer algebra systems such as MAPLE and MATHEMATICA provide suitable software platforms for the development of relevant simulation experiments. Extensive graphics capabilities allow the user to generate two- and three-dimensional graphics, which can be used in the process of mathematical modelling and control. Author concluded that computer algebra systems as MAPLE and MATHEMATICA possess packages for interval arithmetic that can deal with interval arithmetic expressions and are suitable environments for the development of specialized tools. However, these packages are still not very effective in performing symbolic computations involving interval variables and interval arithmetic expressions.

**Mathcad** is mostly oriented to non-programming users, it is also used in more complex projects to visualize results of mathematical modeling by using distributed computing. Unlike MATLAB, the Mathcad offers a rich problem-solving environment just like a pad of pencil and paper with wide choice of tools, supported by a variety of analysis and visualization techniques (**Tutorial on Everyday Mathcad**). It is combination of, a powerful technical computing environment centered on real math notation and a flexible, full-featured technical word processor. Mathcad can create parametric 2D and 3D plot types. It can perform vector and matrix operations, including eigenvalues and eigenvectors. Data can import and export to other applications and file types such as Microsoft Excel and more important that data can integrate with other engineering applications such as Computer Aided Design, Finite Element Method, Building Information Modeling, and Simulation tools, to aid in product design, like AutoCAD, Ansys, and Revit (**Sonaje N P and Waikar M L, 2008**).

**Ioan Neamt et. al. (2014)**, has calculated process parameters using **Mathcad** application, which is a numerical computation mathematical software. Author concluded that calculation summary enables a rapid and accurate calculation of process parameters by using the Mathcad application, which is mathematical computation software. The parameters calculated according to the calculation summary presented, allow obtaining a mass or volume balance, for an co-fermentation process that uses besides sewage sludge and co-substrates with high organic loads.

**Sonaje N. P. and Waikar M. L.(2009)** used **Mathcad** for hydrological analysis using natural resources conservation service (NRCS) method. This study states ease of incorporating Mathcad for the intended application put forth the fact that same is just like undertaking manual analysis using bookish formulae enhanced by technical capabilities of computation owing to customisation and transparency attributes of Mathcad. This facilitates easier automation of analysis and design checking processes. The case study adoption of Mathcad discussed an affordable answer to practical problem of computer efficiency in hydrological analysis.

## SUMMARY OF LITERATURE

After thorough evaluation of the related literature, it can be revealed that most of the work for designing of wastewater treatment plant was carried out by using specially made software using C++ and other languages. These all are black box applications with few advantages and disadvantages too. Certain experiments are done with the treatment processes like Activated Sludge Process; Sequential Batch Reactor, Trickling filters and Membrane Bio Reactor, Moving Bed Bio Film Reactors for treatment of paper industry effluent. With extensive coverage a researcher may go for a suitable and economical method and further tailor the same for obtaining numerical solutions for scientific estimations. The paper has revealed that current design methods of wastewater treatment plants are comprehensive. In general, the usage of mathematical software tools is growing by leaps and bounds. With extensive survey these tools are found to be capable of simulating the design. The review empowers the researchers to choose available technologies and mathematical software will not only improve the design efficiency, but new modeling techniques can be used to design an economical and efficient wastewater treatment plant especially for pulp and paper industry.

## CONCLUSIONS

Above mentioned literature summarised that Mathcad gives user-friendly interface and flexibility in modifying programs at any level with the option to select a particular unit for design, which gives white box approach to all mathematical calculations.

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